A JOINT ECONOMIC-LOT-SIZE MODEL FOR PURCHASER AND VENDOR

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ABSTRACT

In a typical purchasing situation, the issues of price, lot sizing, etc., usually are settled through negotiations between the purchaser and the vendor. Depending on the existing balance of power, the end result of such a bargaining process may be a near-optimal or optimal ordering policy for one of the parties (placing the other in a position of significant disadvantage) or, sometimes, inoptimal policies for both parties. This paper develops a joint economic-lot-size model for a special case where a vendor produces to order for a purchaser on a lot-for-lot basis under deterministic conditions. The focus of this model is the joint total relevant cost. It is shown that a jointly optimal ordering policy, together with an appropriate price adjustment, can be beneficial economically for both parties or, at the least, does not place either at a disadvantage.

Subject Areas: Inventory Management and Production/Operations Management.

INTRODUCTION

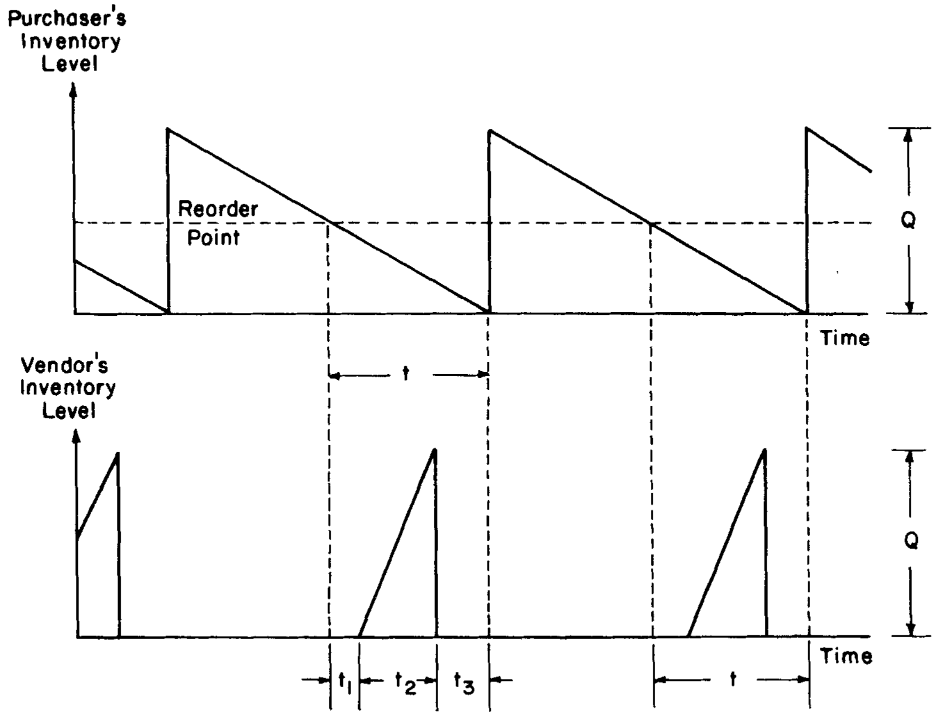
The economic-lot-size (ELS) or the economic-order-quantity (EOQ) formula, attributed to Harris [4] by Hadley and Whitin [3], is a well-known and widely used concept in purchasing and inventory management. Since its inception, the classical ELS formula has been modified and embellished to make it applicable under a variety of conditions (see, e.g., [2] or [6]). Snyder [7] showed the validity of the EOQ model with probabilistic demand; and Jesse, Mitra, and Cox [5], among others, extended its application under inflationary conditions. But a major problem with all these approaches was a failure to take into consideration the overall economic consequences of a system's ELS on other parties.

Consider, for example, a typical purchasing situation where a vendor periodically produces a certain inventory item to order for a purchaser on a lot-for-lot basis. Aside from the question of pricing, one important issue here is that of appropriate lot size. It is obvious that the purchaser's ELS for this item may not result in an optimal policy for the vendor and vice versa. Traditionally, questions of pricing, lot sizing, etc., are settled through negotiation between the two parties. (Buffa [1] presents an excellent discussion of the basic power structure within which such negotiations take place.) More often than not, depending on the existing balance of power, the outcome of such a negotiation results in a near-optimal or optimal policy for one party while the other party is subjected to a substantial cost penalty; in some cases, inoptimal policies result for both parties.

Our aim in this paper is to move away from this adversarial bargaining process and develop the concept of a joint economic-lot-size (JELS) model under deterministic conditions, focusing on the joint total relevant cost (JTRC) for both the buyer and the supplier. We intend to show that a joint optimal policy adopted through a spirit of cooperation can be of economic benefit to both parties. While it may appear initially to involve some cost sacrifice on the part of one party, this can be more than offset by an appropriate price adjustment.

We restrict our discussion and analysis to a relatively simple purchasing scenario. Suppose that a purchaser (buyer) periodically orders some quantity, Q, of an inventory item from a vendor (supplier). With the receipt of an order, the vendor produces the required quantity of the item (i.e., the vendor follows a lot-for-lot policy) and, on completion of the batch, ships the entire lot to the buyer. In addition to deterministic conditions, we assume there are no other buyers for this item and that the vendor in question is the sole supplier. Figure 1 shows the inventory time plots for both parties. The supply lead time, T, consists of three components: t_1 represents the time it takes to transmit a purchase order and set up a production lot, t_2 is the actual production time, and t_3 is the time it takes to deliver the completed lot to the buyer.

FIGURE 1
Purchaser's and Vendor's Inventory Time Plots



It should be noted that the JELS model developed here is limited by the assumptions of the scenario described above. In reality, demand rates and lead times more often than not are stochastic. Furthermore, real-world scenarios are likely to be more complex than the one we outline. For example, in moving toward a just-in-time (JIT) approach, the buyer may opt for a relatively small lot size, and the potential to seek other sources of supply may facilitate this. The vendor may be willing and able to move closer to the buyer's position by lowering his/her setup cost through technological changes. In other cases the vendor, knowing that no other supplier exists for the item in question, may be in a position to take undue advantage of the situation. Some of these issues are discussed in our Conclusions section. In spite of these limitations, our analysis, as an initial research step, is not without value. We hope this work will increase our understanding of some of the intricacies of vendor-buyer cost interactions and that future extensions will incorporate more real-world complexities.

In the following sections we analyze the effects of individual optimization by purchaser and vendor, develop the concept of a JELS model, and discuss its economic implications. Then we present a simple numerical example to illustrate the model and make some concluding remarks.

THE EFFECTS OF INDEPENDENT OPTIMIZATION

For the purposes of this paper, we adopt the following general notation:
D = annual demand or usage of the inventory item,
P = vendor's annual production rate for this item,
A = purchaser's ordering cost per order,
S = vendor's setup cost per setup,
r = annual inventory carrying charge, expressed as a fraction of dollar value,
C_v = unit production cost incurred by the vendor,
C_p = unit purchase cost paid by the purchaser,
Q = order or production lot size in units.
As implied above, for the sake of simplicity we assume that the carrying charge, r, is identical for the buyer and the vendor. Also, note that $P \leq D$ and $C_v \leq C_p$.

Individual Optimal Policies

The derivation of the buyer's or vendor's ELS is relatively simple and well known. The results of individual optimization are summarized in Table 1. Each

TABLE 1
Summary of Relevant Costs and Individual Optimal Policies

	Purchaser		Vendor	
General cost function	$TRC_p(Q)=\frac{DA}{Q}+\frac{Q}{2}rC_p$	(1)	$TRC_v(Q)=\frac{DS}{Q}+\frac{DQ}{2P}rC_v$	(4)
Economic lot size	$Q_p^*=\sqrt{[\frac{2DA}{rC_p}]}$	(2)	$Q_v^*=\sqrt{[\frac{2PS}{rC_v}]}$	(5)
Minimum total cost	$TRC_p(Q_p^*)=\sqrt{[2DArC_p]}$	(3)	$TRC_v(Q_v^*)=D\sqrt{[2SrC_v/P]}$	(6)

Note: $TRC_p(Q)$ = purchaser's annual total relevant cost for any lot size Q, $TRC_v(Q)$ = vendor's annual total relevant cost for any lot size Q, Q_p^* = purchaser's economic lot size (ELS), Q_v^* = vendor's economic lot size (ELS).

party's ELS (result (2) or (5) in Table 1) is obtained by setting the first derivative of the appropriate cost function ((1) or (4)) with respect to Q equal to zero. Then the minimum annual total relevant cost (TRC) for each party ((3) or (6)) results from the substitution of (2) or (5) into (1) or (4), respectively.

The Effects of Purchaser's ELS on Vendor

If the purchaser's ELS is adopted, the vendor's TRC is

$$TRC_v(Q_p^*) = \frac{DS}{\sqrt{[2DA/rC_p]}} + \frac{DrC_v}{2P}\sqrt{[\frac{2DA}{rC_p}]}.$$

Noting that $TRC_v(Q_v^*) = D\sqrt{[2SrC_v/P]}$ and defining $\alpha = S/A$ and $\beta = DC_v/PC_p$, the above equation simplifies to

$$TRC_v(Q_p^*) = \frac{\frac{1}{2}(\alpha+\beta)}{\sqrt{(\alpha\beta)}}\,TRC_v(Q_v^*). \tag{7}$$

As mentioned earlier, for the sake of simplicity we assume that r (the inventory carrying charge), expressed as a fraction of dollar value, is the same for the buyer and the vendor. In reality, the carrying charges for the two parties often may differ. But our assumption does not lead to any loss in generality. If in fact the purchaser's and the vendor's carrying charges, denoted by r_p and r_v, respectively, are unequal, the parameter β is redefined more generally as Dr_vC_v/Pr_pC_p and all the major results obtained here essentially remain unchanged.

Physical interpretations of α and β may help us describe and understand the practical implications of the various conditions associated with these parameters. As defined earlier, $\alpha = S/A$, that is, α represents the ratio of the vendor's setup cost per setup (S) to the purchaser's ordering cost per order (A). Also, in (1) in Table 1, the purchaser's total annual setup cost is DA/Q and from (4) the vendor's total annual setup cost is DS/Q. We then can show that $(DS/Q)/(DA/Q) = S/A = \alpha$. In other words, α can also be interpreted as the ratio of the vendor's total annual (or periodic) setup cost to the purchaser's total annual (or periodic) ordering cost for any given lot size. Thus, when the vendor's setup cost is large compared to the buyer's ordering cost, α tends to be large and vice versa.

In a similar vein, the vendor's total annual inventory carrying cost from (4) is $DQrC_v/2P$ and from (1) the buyer's total annual inventory carrying cost is $QrC_p/2$. In the general case, when $r_v \neq r_p$,

$$(DQr_vC_v/2P)/(Qr_pC_p/2) = Dr_vC_v/Pr_pC_p = \beta.$$

When $r_v = r_p = r$, $DC_v/PC_p = \beta$. Thus β represents the ratio of the vendor's total annual (or periodic) carrying cost to the purchaser's total annual (or periodic) carrying cost for any given lot size. Note that when the vendor's production rate, P, is high compared to the demand rate, D, the value of β tends to be relatively small.

In practical terms, a high vendor production rate requires less time to produce a given lot size (i.e., the interval t_2 in Figure 1 tends to be smaller). As a result of this, the vendor's annual or periodic inventory carrying cost tends to be low and, consequently, results in a relatively low value for β.

By the same token, the parameter β also can be small if the vendor's unit carrying (or variable production) cost is low compared to the buyer's unit carrying (or purchase) cost. Therefore, it is reasonable to surmise that a high production rate compared to demand and/or a relatively high unit carrying cost on the part of the buyer tends to lower the value of β and vice versa.

Using the result from equation (7), we now can express the vendor's percentage cost penalty (*PCP*) as

$$PCP_v(Q_v^* \rightarrow Q_p^*) = \frac{[\frac{1}{2}(\alpha+\beta)/\sqrt{(\alpha\beta)}]TRC_v(Q_v^*) - TRC_v(Q_v^*)}{TRC_v(Q_v^*)}(100).$$

(The notation $PCP_v(Q_v^* \rightarrow Q_p^*)$ denotes the vendor's percentage cost penalty for producing and supplying the purchaser's ELS, Q_p^*, instead of the vendor's ELS, Q_v^*.) This expression simplifies to

$$PCP_v(Q_v^* \rightarrow Q_p^*) = [\tfrac{1}{2}(\alpha+\beta)/\sqrt{(\alpha\beta)} - 1](100). \tag{8}$$

Also, the vendor's absolute cost penalty (*ACP*) resulting from the purchaser's ELS is

$$ACP_v(Q_v^* \rightarrow Q_p^*) = [\tfrac{1}{2}(\alpha+\beta)/\sqrt{(\alpha\beta)} - 1]TRC_v(Q_v^*). \tag{9}$$

FIGURE 2
Vendor's (Purchaser's) *PCP* if Purchaser (Vendor) Optimizes

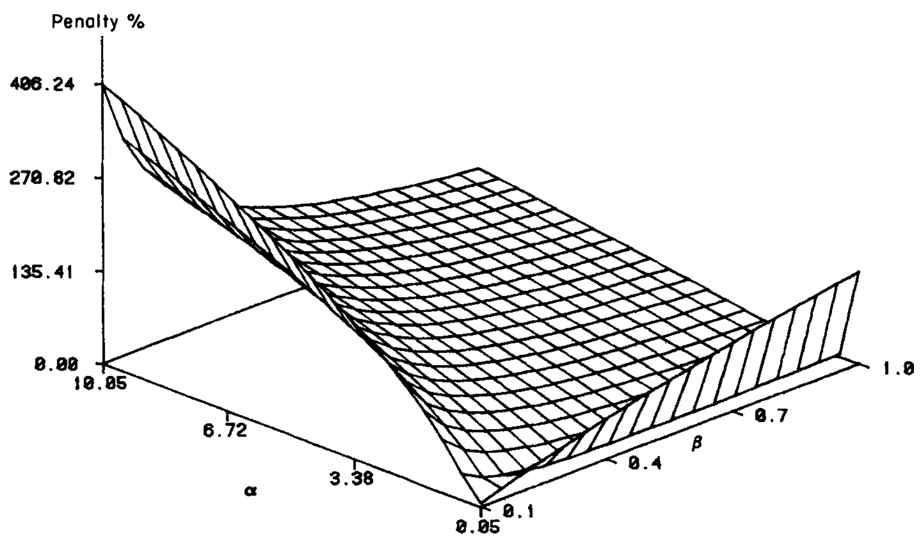


Figure 2 shows a three-dimensional plot of the vendor's *PCP* against the two parameters α and β. Usually S, the vendor's setup cost per setup, is likely to be greater than A, the purchaser's ordering cost per order, but not necessarily so; this implies that $0<\alpha<\infty$. Also, as noted earlier, $P \geq D$ and $C_v \leq C_p$; therefore, $0<\beta \leq 1$. From Figure 2 it is clear that the vendor's *PCP* or *ACP* can be substantial—particularly for a large α in conjunction with a small β—as a result of adopting the buyer's ELS. This means that when the vendor's setup cost is very much larger than the buyer's ordering cost and the carrying-cost ratio is small, his/her ELS tends to be significantly larger than the purchaser's. Thus, adoption of the buyer's ELS forces the vendor to have too frequent setups, resulting in a high cost penalty.

On the other hand, Figure 2, as well as equations (8) and (9), indicates that the vendor's cost penalty approaches zero as α approaches β. This implies that when the fixed-cost ratio approaches the carrying-cost ratio the vendor's ELS gets smaller and approaches the purchaser's ELS. Thus, adoption of the buyer's ELS results in a smaller cost penalty on the part of the vendor. When the two cost ratios are equal ($\alpha=\beta$), the vendor's ELS is the same as the buyer's and the vendor's cost penalty is zero. In other words, the two cost ratios offset each other and the two parties' optimal policies coincide.

The Effects of Vendor's ELS on Purchaser

Suppose the vendor's ELS is the order quantity. Then, by substituting the result of equation (5) into equation (1), we can show that the purchaser's *TRC* is

$$TRC_p(Q_v^*) = [\tfrac{1}{2}(\alpha+\beta)/\sqrt{(\alpha\beta)}]\,TRC_p(Q_p^*). \tag{10}$$

Again, using this result, the purchaser's *PCP* for adopting the vendor's ELS instead of his/her own can be shown to be

$$PCP_p(Q_p^* - Q_v^*) = [\tfrac{1}{2}(\alpha+\beta)/\sqrt{(\alpha\beta)}-1](100). \tag{11}$$

The purchaser's *ACP* is

$$ACP_p(Q_p^* \rightarrow Q_v^*) = [\tfrac{1}{2}(\alpha+\beta)/\sqrt{(\alpha\beta)}-1]\,TRC_p(Q_p^*). \tag{12}$$

Note that the right-hand sides of (8) and (11) are identical and that (9) and (12) are very similar in form, that is, the vendor's *PCP* for adopting the purchaser's ELS is identical to the purchaser's *PCP* for adopting the vendor's ELS. Thus, Figure 2 also shows the purchaser's *PCP* for adopting the vendor's ELS against α and β.

From (2) and (5) and the definitions of α and β, we can show that:

$$Q_v^* = \sqrt{(\alpha/\beta)}\,Q_p^* \tag{13}$$

and from (3) and (6)

$$TRC_v(Q_v^*) = \sqrt{(\alpha\beta)}TRC_p(Q_p^*). \tag{14}$$

From (13) we see that when $\alpha > \beta$ (i.e., when the two parties' fixed-cost ratios exceed their carrying-cost ratios) $Q_p^* < Q_v^*$. In other words, when the purchaser's ordering cost is relatively low and his/her carrying cost is relatively high, Q_p^* (his/her own ELS) is less than Q_v^* (the vendor's ELS). Under these conditions, adopting the vendor's ELS forces the buyer to order larger quantities less frequently, thus increasing his/her average inventory and annual total cost. On the other hand, $\alpha < \beta$ implies that the purchaser's ordering cost is relatively high and his/her carrying cost is relatively low, which is unlikely (but not impossible) to occur in practice. In this case, Q_p^* (the buyer's ELS) is larger than Q_v^* (the vendor's ELS). Now if the vendor's ELS is adopted, the buyer is forced to order smaller quantities more frequently, which increases his/her annual ordering as well as total costs.

In more general terms, as the difference between α and β grows larger, the adoption of either party's ELS places the other party farther away from his/her own optimal position, thus increasing his/her cost penalty. By the same token, as the difference between α and β grows smaller, the optimal positions of the buyer and the vendor (in terms of their respective lot sizes) draw closer. When $\alpha = \beta$ (i.e., when any disparity in the two parties' fixed costs is offset exactly by a similar disparity in their carrying costs *in the same direction*), their optimal lot sizes are identical. Obviously, in such a circumstance the adoption of one party's ELS will not result in any penalty for the other party.

Theoretically, the condition $\alpha = \beta$ can occur in a variety of situations. A value of β close to unity (recall that $0 < \beta \leq 1$) implies that the vendor's production rate is close to the demand rate and also that the buyer's and vendor's unit carrying costs are similar. In such a case, the condition $\alpha = \beta$ is approached when the vendor's fixed cost per setup is similar to the buyer's fixed cost per order. On the other hand, when β is much smaller than 1, the above condition is approached when the buyer's ordering cost is much larger than the vendor's setup cost. Neither of these scenarios, however, commonly occurs in the real world. Under most practical conditions, the adoption of either party's optimal policy places the other in a position of serious disadvantage.

From equation (14), note that even if α and β are equal the vendor's minimum *TRC* and the buyer's minimum *TRC* are not necessarily the same (unless, of course, $\alpha\beta = 1$). Since $\beta \leq 1$, $\alpha\beta = 1$ implies that $\alpha \geq 1$. Thus, to the extent the vendor's relatively high fixed cost is counterbalanced by a relatively low unit-carrying cost and/or a high production rate compared to demand, his/her *TRC* resulting from his/her optimal policy approaches that of the purchaser. In other words, the condition $\alpha\beta = 1$ indicates that any imbalance in the two parties' fixed costs is exactly compensated by a similar imbalance in their carrying costs *in the opposite direction.* In such a case, their *TRC*s (which result from their respective optimal policies) are exactly equal. In the real world, this is much more likely to occur than the situation where the two parties' optimal lot sizes are the same (i.e., where $\alpha = \beta$).

A JELS MODEL

We have outlined the drawbacks for each party resulting from the adoption of the other's ELS. We now turn to the derivation of a JELS model. For the buyer and the vendor, the joint *TRC* (*JTRC*) for any lot size Q is produced by adding equations (1) and (4) as follows:

$$JTRC(Q) = {}^{D}\!/_{Q}(S+A) + {}^{Q}\!/_{2}r({}^{D}\!/_{P}C_v + C_p). \tag{15}$$

By setting the first derivative of this cost function with respect to Q equal to zero at $Q = Q_J^*$, we obtain the JELS:

$$Q_J^* = \sqrt{\left[\frac{2D(S+A)}{r({}^{D}\!/_{P}C_v + C_p)}\right]}. \tag{16}$$

Substituting Q_J^* in (15), the minimum *JTRC* per year is

$$JTRC(Q_J^*) = \sqrt{[2Dr(S+A)({}^{D}\!/_{P}C_v + C_p)]}. \tag{17}$$

Equation (16) may be rewritten as

$$Q_J^* = \sqrt{[(1+\alpha)/(1+\beta)]}Q_p^*. \tag{18}$$

Also, from (13) and (18),

$$Q_J^* = \sqrt{[(1+{}^{1}\!/_{\alpha})/(1-{}^{1}\!/_{\beta})]}Q_v^*. \tag{19}$$

Similarly, we can show that

$$JTRC(Q_J^*) = \sqrt{[(1+\alpha)(1+\beta)]}\, TRC_p(Q_p^*) \tag{20}$$

and

$$JTRC(Q_J^*) = \sqrt{[(1+{}^{1}\!/_{\alpha})(1+{}^{1}\!/_{\beta})]}\, TRC_v(Q_v^*). \tag{21}$$

Equations (18) and (19) above express, respectively, the relationships between the JELS and the ELS of the purchaser and the vendor. Equations (20) and (21), on the other hand, show the relationships between the optimal *JTRC* and the purchaser's and vendor's individual optimal total costs, respectively.

Examining (18) and (19), it is obvious that if $\alpha = \beta$, $Q_J^* = Q_p^* = Q_v^*$. On the other hand, if $\alpha > \beta$, $Q_p^* < Q_J^* < Q_v^*$; if $\alpha < \beta$, $Q_v^* < Q_J^* < Q_p^*$. Thus, the JELS represents a compromise between the buyer's ELS and the vendor's ELS when they are unequal, which usually is the case. This likely imbalance points out the need for such a compromise, in most cases, in order to reduce the *JTRC*. We show later that adoption of the JELS, accompanied by an appropriate price adjustment, can indeed be beneficial for both parties.

Joint Economic Consequences of Individual Optimization

Before discussing the economic effects of the JELS, we derive the *JTRC*s for the purchaser's and the vendor's optimal policies. If the purchaser's ELS is adopted, the *JTRC* is expressed by

$$JTRC(Q_p^*) = [1 + \tfrac{1}{2}(\alpha+\beta)]TRC_p(Q_p^*), \tag{22}$$

which is derived from equations (7) and (14). Similarly, using relationships (10) and (14), the *JTRC* for adopting the vendor's ELS is

$$JTRC(Q_v^*) = [1 + \tfrac{1}{2}(\tfrac{1}{\alpha} + \tfrac{1}{\beta})]TRC_v(Q_v^*). \tag{23}$$

If the order quantity is the purchaser's ELS instead of the JELS, the joint *PCP* (*JPCP*) is given by

$$JPCP(Q_J^* \rightarrow Q_p^*) = [\frac{JTRC(Q_p^*) - JTRC(Q_J^*)}{JTRC(Q_J^*)}](100).$$

Using (20) and (22), we then can show that

$$JPCP(Q_J^* \rightarrow Q_p^*) = [\frac{1 + \tfrac{1}{2}(\alpha+\beta)}{\sqrt{[(1+\alpha)(1+\beta)]}} - 1](100). \tag{24}$$

Similarly, using relationships (21) and (23), we obtain the *JPCP* for adopting the vendor's ELS instead of the JELS. That is,

$$JPCP(Q_J^* \rightarrow Q_v^*) = [\frac{1 + \tfrac{1}{2}(\tfrac{1}{\alpha} + \tfrac{1}{\beta})}{\sqrt{[(1+\tfrac{1}{\alpha})(1+\tfrac{1}{\beta})]}} - 1](100). \tag{25}$$

Figure 3, which graphically represents the *JPCP* for the purchaser's optimal policy, shows that the joint penalty can be substantial, particularly for a high α and a low β. This is not surprising since a high α and a low β translate, respectively, to a high setup cost and a low carrying cost for the vendor. Thus, with the purchaser's ELS in effect, the vendor has too-frequent setups, contributing to the *JTRC* and the joint penalty. The joint *ACP* (*JACP*) in this situation can be determined from equations (20) and (22). That is,

$$JACP(Q_J^* \rightarrow Q_p^*) = \tfrac{1}{2}(\sqrt{(1+\alpha)} - \sqrt{(1+\beta)})^2 TRC_p(Q_p^*). \tag{26}$$

In accordance with Figure 3, this joint cost penalty vanishes for $\alpha = \beta$, that is, when the setup-cost and carrying-cost ratios are equal.

FIGURE 3

***JPCP* for Adopting Purchaser's ELS or Purchaser's *PCP* for Adopting JELS**

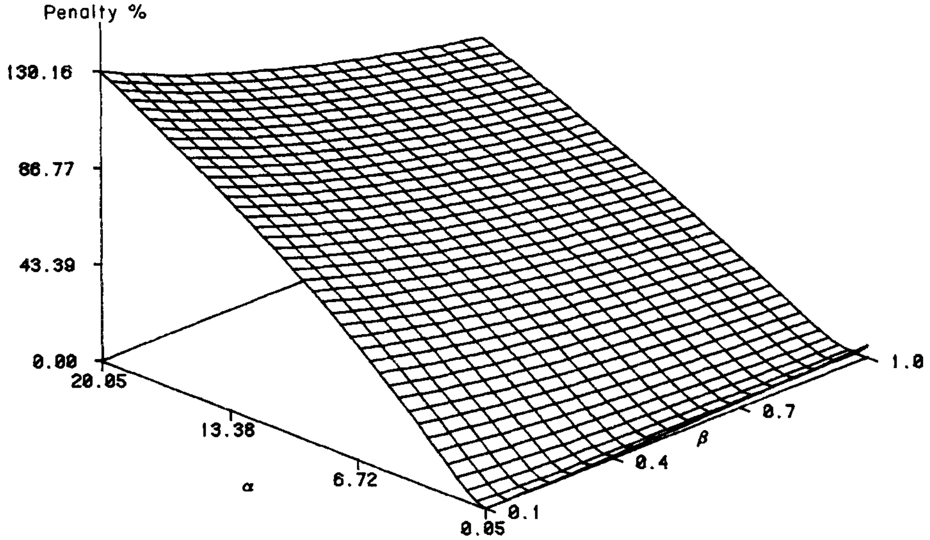


FIGURE 4

***JPCP* for Adopting Vendor's ELS or Vendor's *PCP* for Adopting JELS**

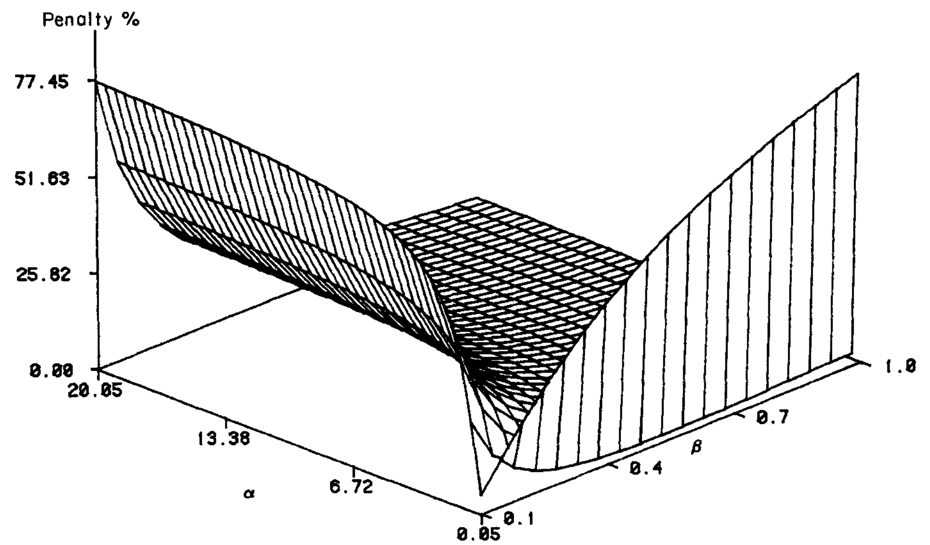


Figure 4 is a three-dimensional plot of the *JPCP* for adopting the vendor's ELS. Again, this plot shows that the *JPCP* increases with increasing α and decreasing β. One difference between this plot and Figure 3 is that for a given β the *JPCP* appears to level off beyond an α of 10 or so and is less severe (compared to the previous case when both α and β were high). Also, the joint penalty appears to increase dramatically when α is less than β.

As before, using (21) and (23), the *JACP* for adopting the vendor's ELS instead of the JELS is

$$JACP(Q_J^* \rightarrow Q_v^*) = \tfrac{1}{2}(\sqrt{(1+1/\alpha)} - \sqrt{(1+1/\beta)})^2 TRC_v(Q_v^*). \quad (27)$$

Again, as expected, this joint cost penalty is zero when the parameters α and β are equal; the practical implications of this have been discussed.

Individual Economic Consequences of the JELS

In order to ascertain the individual economic consequences of adopting the JELS, we need to derive the buyer's and the vendor's *TRC*s which would result. Substituting (18) into (1) and (19) into (4), respectively, we obtain

$$TRC_p(Q_J^*) = \left[\frac{1+\tfrac{1}{2}(\alpha+\beta)}{\sqrt{[(1+\alpha)(1+\beta)]}}\right] TRC_p(Q_p^*) \quad (28)$$

and

$$TRC_v(Q_J^*) = \left[\frac{1+\tfrac{1}{2}(1/\alpha+1/\beta)}{\sqrt{[(1+1/\alpha)(1+1/\beta)]}}\right] TRC_v(Q_v^*). \quad (29)$$

These are used for determining the economic effects of the JELS on each party. Table 2 summarizes these effects.

If the JELS is adopted instead of the purchaser's ELS (Q_p^*), the purchaser's *ACP* and *PCP* (results (30) and (31), respectively, in Table 2) are obtained directly from (28). In this situation, the vendor's absolute- and percentage-cost advantages (*ACA* and *PCA*, respectively, (36) and (37) in Table 2), are derived from (7) and (29). Similarly, if the JELS is adopted instead of the vendor's ELS (Q_v^*), the buyer's *ACA* and *PCA* ((32) and (33)) result from (10) and (28). The vendor's *ACP* and *PCP* in this case ((34) and (35)) are obtained from (29).

It is interesting to note that the right-hand sides of (31) and (24) are identical, implying that the purchaser's individual *PCP* for adopting Q_J^* is the same as the *JPCP* for both parties if the lot size is Q_p^* instead of Q_J^*. Also, equations (35) and (25) have identical right-hand sides, indicating that the vendor's *PCP* for adopting the JELS is the same as the *JPCP* if the lot size is Q_v^* instead of Q_J^*. Thus, Figures 3 and 4 also are plots, respectively, of the buyer's and the vendor's individual *PCP*s resulting from joint rather than individual optimization. In addition, Figures 5 and 6 graphically depict the vendor's and the buyer's *PCA*s resulting from the adoption of the JELS instead of the other party's ELS. From these figures, it is apparent that if the ELS of one of the parties currently is in effect, the other party stands to gain significantly from adopting the JELS, particularly when α is large or when β exceeds α.

TABLE 2
Summary of Individual Economic Consequences of the JELS

	Lot Size Changed from Q_p^* to Q_J^*		Lot Size Chang
Purchaser's penalty	$ACP_p(Q_p^* \to Q_J^*) = [\frac{1 + \frac{1}{2}(\alpha + \beta)}{\sqrt{[(1+\alpha)(1+\beta)]}} - 1] TRC_p(Q_p^*)$	(30)	None
	$PCP_p(Q_p^* \to Q_J^*) = [\frac{1 + \frac{1}{2}(\alpha + \beta)}{\sqrt{[(1+\alpha)(1+\beta)]}} - 1](100)$	(31)	
Purchaser's advantage	None		$ACA_p(Q_v^* \to Q_J^*) = [1 - \frac{1+2/}{\sqrt{[(1+ ^1/}}$
			$PCA_p(Q_v^* \to Q_J^*) = [1 - \frac{1+2/}{\sqrt{[(1+ ^1/}}$
Vendor's penalty	None		$ACP_v(Q_v^* \to Q_J^*) = [\frac{1 + \frac{1}{2}(^1/\alpha}{\sqrt{[(1+ ^1/\alpha)(}}$
			$PCP_v(Q_v^* \to Q_J^*) = [\frac{1 + \frac{1}{2}(^1/\alpha -}{\sqrt{[(1+ ^1/\alpha)(1}}$
Vendor's advantage	$ACA_v(Q_p^* \to Q_J^*) = [1 - \frac{1 + 2/(^1/\alpha + ^1/\beta)}{\sqrt{[(1+\alpha)(1+\beta)]}}] TRC_v(Q_p^*)$	(36)	None
	$PCA_v(Q_p^* \to Q_J^*) = [1 - \frac{1 + 2/(^1/\alpha + ^1/\beta)}{\sqrt{[(1+\alpha)(1+\beta)]}}](100)$	(37)	

Note: ACP = absolute cost penalty, ACA = absolute cost advantage, PCP = percentage cost penalty, PCA = percenta

FIGURE 5
Vendor's *PCA* for Lot Size of Q_J^* Instead of Q_p^*

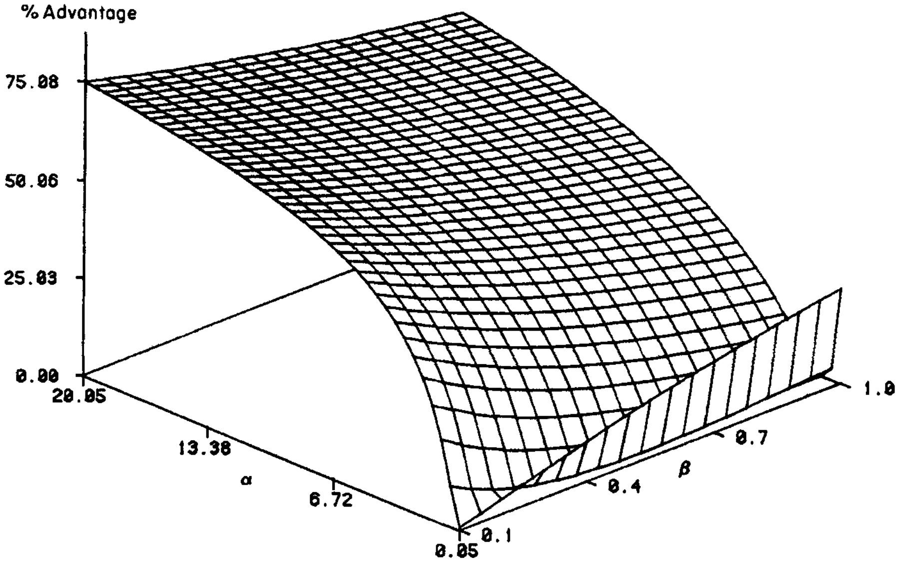
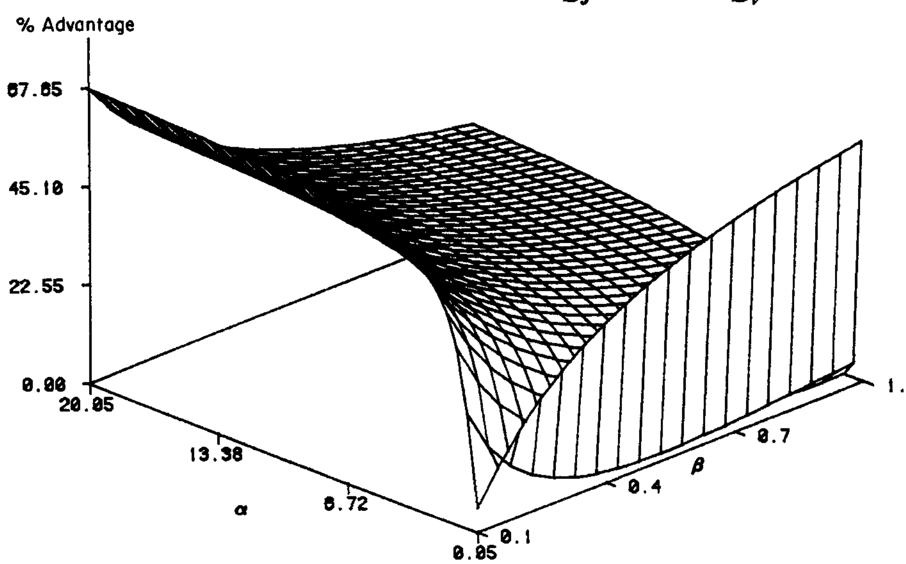


FIGURE 6
Purchaser's *PCA* for Lot Size of Q_J^* Instead of Q_v^*



Cost Trade-Offs and Price Adjustments

We can examine the cost trade-offs associated with joint optimization from one of two perspectives. First, if the JELS is adopted, rather than the purchaser's

ELS, the purchaser incurs a loss ($ACP_p(Q_p^* \rightarrow Q_J^*)$ given by (30)), but the vendor gains an amount ($ACA_v(Q_p^* \rightarrow Q_J^*)$ given by (36)). We know that the *JTRC* for a lot size of Q_J^* cannot be greater than that resulting from a lot size of Q_p^*. So it follows logically that the purchaser's loss is more than offset by the vendor's gain ($ACA_v(Q_p^* \rightarrow Q_J^*) > ACP_p(Q_p^* \rightarrow Q_J^*)$) and the joint *ACA* (*JACA*) in this case is obtained by subtracting the latter from the former. More easily, by definition the *JACA* resulting from Q_J^* rather than Q_p^* is the same as the *JACP* for adopting Q_p^* instead of Q_J^*. That is, from (26)

$$JACA(Q_p^* \rightarrow Q_J^*) = \tfrac{1}{2}(\sqrt{(1+\alpha)} - \sqrt{(1+\beta)})^2 TRC_p(Q_p^*). \tag{38}$$

To entice the buyer to change his/her lot size from Q_p^* to Q_J^*, the vendor may offer a unit price discount of d, for which the upper and lower bounds are set by

$$\tfrac{1}{D}[ACP_p(Q_p^* \rightarrow Q_J^*) \leq d \leq \tfrac{1}{D}[ACA_v(Q_p^* \rightarrow Q_J^*)], \tag{39}$$

that is,

$$d_{\min} \leq d \leq d_{\max}. \tag{40}$$

If d is set at $d_{\min}$, all benefits resulting from the adoption of the JELS accrue to the vendor and the purchaser is indifferent between Q_p^* and Q_J^*. If, on the other hand, d is set at $d_{\max}$, all benefits of joint optimization go to the buyer and the vendor's *TRC* remains unchanged. Perhaps an equitable or "fair" way to allocate the joint benefits would be to divide the *JACA* as a result of changing the lot size from Q_p^* to Q_J^*, given by (38), equally between the vendor and the purchaser. In other words, let

$$d = \tfrac{1}{4}(\sqrt{(1+\alpha)} - \sqrt{(1+\beta)})^2 \frac{TRC_p(Q_p^*)}{D}, \tag{41}$$

for which both parties benefit equally from the JELS.

From the second perspective, the vendor's ELS is in effect and the purchaser wishes to change this to the JELS. In order to entice the vendor to do so, the buyer offers a unit purchase price increase of u. As a result of this lot-size change, the vendor's loss would be $ACP_v(Q_v^* \rightarrow Q_J^*)$ (obtained from (34)) and the purchaser would gain an amount $ACA_p(Q_v^* \rightarrow Q_J^*)$ (shown by (32)). Using equation (27), the *JACA* in this case is

$$JACA(Q_v^* \rightarrow Q_J^*) = \tfrac{1}{2}(\sqrt{(1+1/\alpha)} - \sqrt{(1+1/\beta)})^2 TRC_v(Q_v^*). \tag{42}$$

As before, the bounds on u (the upward price adjustment per unit) are set by

$$\tfrac{1}{D}[ACP_v(Q_v^* \rightarrow Q_J^*)] \leq u \leq \tfrac{1}{D}[ACA_p(Q_v^* \rightarrow Q_J^*)]. \tag{43}$$

That is,

$$u_{\min} \leq u \leq u_{\max}. \tag{44}$$

When $u = u_{\min}$, all the benefits derived from adopting the JELS go to the purchaser; and when $u = u_{\max}$, all the benefits accrue to the vendor. Again, perhaps a "fair" unit price increase may be obtained by dividing the net joint benefits equally among the two parties. That is, using (42), let

$$u = \tfrac{1}{4}(\sqrt{(1+1/\alpha)} - \sqrt{(1+1/\beta)})^2 \frac{TRC_v(Q_v^*)}{D}. \tag{45}$$

This would produce an equal gain for both parties for adopting Q_J^* instead of Q_v^*.

It may be argued that, when the party at a disadvantage offers a price adjustment to lure the other party to adopt the JELS, the latter may recompute his/her ELS using the new price and adopt this instead of the JELS. To address this argument in detail, we need to examine four separate cases involving the two perspectives and the relative magnitudes of α and β.

For the sake of brevity, in Figure 7 we illustrate only one of these cases where the buyer's ELS is in effect and $\alpha > \beta$. From Figure 7 it is clear that, if the vendor offers a price discount d, the JELS represents a break point in the buyer's cost function and is always his/her optimal order quantity as long as $d_{\min} \leq d \leq d_{\max}$. (In the limiting case where $d = d_{\min}$, the buyer is indifferent between Q_p^* and Q_J^*.) In this and every other case, we can prove easily that as long as the price adjustment offered by the party at a disadvantage is within the bounds expressed by (39) or (43), the JELS is the optimal lot size for the other party.[1]

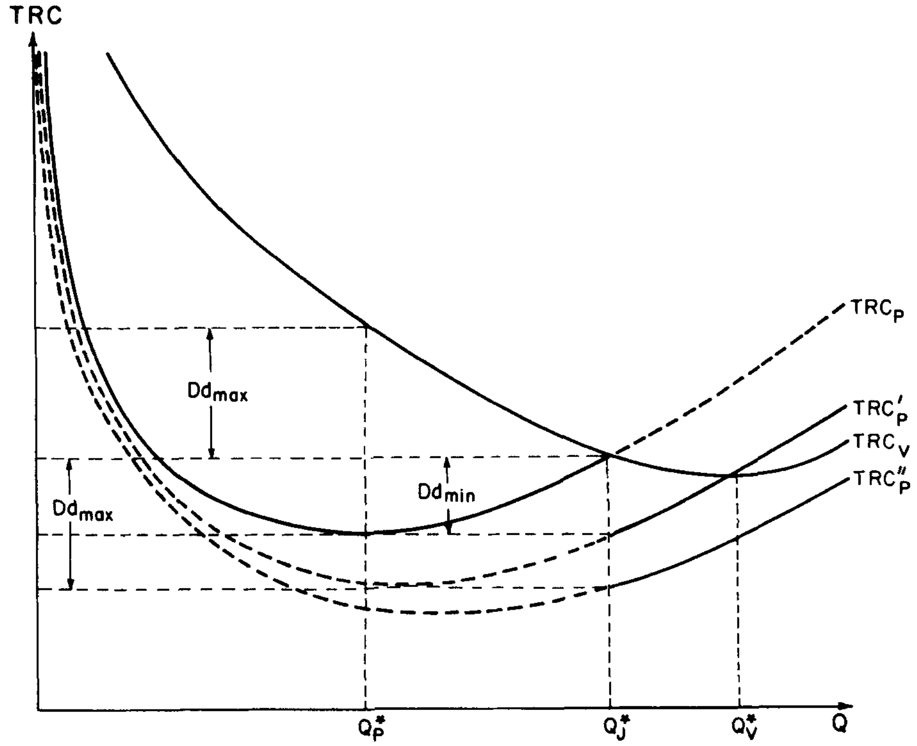
In view of the above discussion, it is clear that under all circumstances the party at a disadvantage can make a persuasive argument in favor of adopting the JELS in conjunction with an appropriate price adjustment. Adoption of the JELS in lieu of any other lot size can benefit both the purchaser and the vendor without the need for cost sacrifice on the part of either party. A numerical example presented in the next section illustrates the cost trade-offs and price adjustments discussed here.

A NUMERICAL EXAMPLE

Consider the case of an inventory item produced to order by a vendor on a lot-for-lot basis. A single purchaser periodically orders and buys a batch of this item from the vendor, who is the buyer's sole source for this item. The following parameters are known: $D = 1000$ units/year, $P = 3200$ units/year, $A = \$100$/order, $S = \$400$/setup, $C_p = \$25$/unit, $C_v = \$20$/unit, and $r = .2$. Thus, $\alpha = 400/100 = 4.0$ and $\beta = (1000 \times 20)/(3200 \times 25) = .25$.

[1]A rigorous mathematical proof of this for each case and a detailed discussion of all four cases are available from the author on request.

FIGURE 7
Cost Curves for Changing Lot Size from Q_p^* to Q_J^*
(Case 1: $\alpha > \beta$, Price Discount Applies iff $Q_p \geq Q_J^*$)



TRC_p = purchaser's total relevant cost curve without price discount, TRC_v = vendor's total relevant cost curve, TRC_p' = purchaser's total relevant cost curve with a unit price discount of $d_{\min}$, TRC_p'' = purchaser's total relevant cost curve with a unit price discount of $d_{\max}$.

The purchaser's ELS is: $Q_p^* = \sqrt{[(2(1000)(100))/(.2(25))]} = 200$ units. The resulting *TRC* for the buyer is $TRC_p(Q_p^*) = \sqrt{[2(1000)(100)(.2)(25)]} = \$1000$. Similarly, using (13) and (14), the vendor's ELS and the resulting minimum *TRC*, respectively, are: $Q_v^* = \sqrt{(4/.25)}(200) = 800$ units and $TRC_v(Q_v^*) = \sqrt{[(4)(.25)]}(1000) = \$1000$, which also may be computed using (5) and (6), respectively.

If the purchaser's ELS of 200 units is the order quantity, the vendor's *TRC*, according to equation (7), is

$$TRC_v(Q_p^*) = \frac{.5(4+.25)}{\sqrt{[(4)(.25)]}}(1000) = \$2125.$$

In this case the vendor's anual *ACP* is \$1125, which is 112.5 percent of his/her minimum *TRC*.

On the other hand, if the vendor's ELS of 800 units is adopted, from equation (10) the purchaser's *TRC* is

$$TRC_p(Q_v^*) = \frac{.5(4+.25)}{\sqrt{[(4)(.25)]}}(1000) = \$2125,$$

which also represents a 112.5 percent or \$1125-per-year cost penalty for the buyer. Thus, if either party optimizes, the other party is at a significant disadvantage; the resulting annual *JTRC* is \$3125. Note that in this example, $TRC_p(Q_p^*) = TRC_v(Q_v^*)$ since $\alpha\beta = 1$.

We may find the JELS using any one of equations (17), (18), or (19). For example, from (18)

$$Q_j^* = \sqrt{[\frac{1+4}{1+.25}]}(200) = 400 \text{ units}$$

and the resulting *JTRC* from equation (20) is

$$JTRC(Q_j^*) = \sqrt{[(1+4)(1+.25)]}(1000) = \$2500.$$

Therefore, the result of adopting either the buyer's or the vendor's ELS is a *JACP* of \$625 per year (i.e., a 25 percent *JACP*).

If the JELS is the order quantity, from (28) the purchaser's *TRC* is

$$TRC_p(Q_j^*) = \frac{1+.5(4+.25)}{\sqrt{[(1+4)(1+.25)]}}(1000) = \$1250.$$

This implies an annual *TRC* of \$1250 for the vendor as well, which can be verified by equation (29).

Suppose the purchaser's ELS (200 units) is in effect. Adopting the JELS (400 units) would result in a cost penalty of \$250 (i.e., 25 percent) per year on the part of the purchaser. But at the same time, the vendor saves \$875 (i.e., about 41 percent annually). Thus the purchaser's loss is more than offset by the vendor's gain. As seen earlier, the net *JACA* resulting from the JELS is \$625 a year. If the vendor now offers a price discount of 25 cents per unit and persuades the buyer to change the lot size from 200 to 400 units, the \$250 annual increase in the purchaser's ordering and carrying costs is offset by the savings (\$.25 × 1000) in the item's annual purchase cost.

In order to be "fair" (i.e., if the joint cost savings of \$625 a year is to be shared equally by both parties), the price discount may be 31.25 cents per unit (625/2(1000)). In this case, the vendor's net gain would be \$312.50 per year and the buyer would gain \$75 a year (i.e., \$312.50 less \$250 plus a \$12.50 reduction in carrying cost resulting from the new C_p of \$24.6875) compared to the buyer's independent optimal ordering policy with the old C_p of \$25 in effect. It should be noted that, strictly speaking, any change in the item's purchase price would result in a JELS different from the 400 units based on a C_p of \$25/unit. For most practical purposes, however, the changes in the lot size and the carrying costs are likely to be small and can be ignored safely. For example, from equation (16) the exact JELS for $C_p = \$24.6875$ is about 402 units. But adopting the old JELS of 400 units, based on $C_p = \$25$ instead of the adjusted JELS, will result in approximately a .0013 percent increase in the joint *TRC*, which is negligible. (See, e.g., [6] for a sensitivity analysis of the classical ELS which also is valid for the JELS.)

Similarly if the vendor's ELS (800 units) is the current order quantity, adoption of the JELS would result in a cost increase of \$250 a year on the part of the vendor but a savings of \$875 a year for the buyer, that is, a net joint total savings of \$625 annually. Using a line of argument similar to the one presented above, the purchaser now may offer the vendor an appropriate increase in the unit purchase price, at least offsetting the latter's cost increase, for adopting the JELS. If, on the other hand, the current lot size is neither Q_p^* nor Q_v^*, an appropriate price adjustment can be made to persuade the party for whom joint optimization results in a cost increase to adopt the JELS. It is ironic that in some situations the vendor may request a price cut or the buyer may desire a price increase; this is contrary to current practice in most traditional bargaining processes. But, as pointed out above, the economic justifications of such a phenomenon are sound from the standpoint of either party.

CONCLUSIONS

In most purchasing situations the issues of price, lot sizing, etc., are settled by negotiation between the purchaser and the vendor. Such a bargaining process often results in an advantageous cost or profit position for one of the parties, while the other may be at a significant disadvantage. This paper has dealt with the special case of a single purchaser and a single vendor. The latter produces an inventory item to order and is the sole supplier of the item; there are no other buyers for the item. In the context of this scenario, we analyzed the effects of each party's optimal lot size on the other and developed a JELS model that focused on the joint total relevant cost (*JTRC*). We realize, however, that a real-world vendor may have an advantage over the purchaser unless the purchaser can develop other sources of supply or can keep the information that there are no other suppliers from the vendor.

We demonstrated the advantage of the JELS approach through an analysis of the cost trade-offs (in conjunction with an appropriate price adjustment) from the perspective of each party's optimal position. Our major conclusion is that by adopting a jointly optimal ordering policy, one party's loss is more than offset by the gain of the other, and the net benefit can be shared by both parties in some equitable fashion. We hope this finding will underscore the necessity and desirability of cooperation among buyers and vendors in establishing purchasing and/or supply policies and thereby contribute to better purchaser-vendor relations.

As Buffa [1] pointed out, in order to compete effectively in the world market many U.S. manufacturers may have to adopt just-in-time (JIT) concepts developed in Japan. Indeed, a number of organizations in this country, particularly in the automobile and related industries, already are moving in this direction [1]. An important implication of this development is that for those manufacturers who successfully adopt such concepts, production setup costs and times eventually will be reduced so that the major benefits of JIT systems (i.e., lower inventory levels and the flexibility to change over from one product to another with ease) can be maximized with the ideal lot size of one.

In this sense, perhaps the idea of large setup and ordering costs some day may become obsolete. Nevertheless, most U.S. firms are several years away from attaining such an ideal; in the interim, they may have to cope with sizable ordering and setup efforts. Therefore, our JELS model can be viewed as at least an intermediate step toward the shift to JIT techniques. Moreover, successful implementation of JIT concepts would require a new spirit of cooperation between purchasers and vendors. As observed above, our suggested approach may be helpful in breaking down the traditional barriers and as such be a first step toward creating such a climate.

The primary limitation of our approach (and of any other deterministic inventory analysis) is the assumption of deterministic demand and lead times. In reality, both of these may be random variables. In addition to the incorporation of stochastic considerations, we can recommend for possible extensions of this research the following cases: a single vendor and multiple purchasers, a single purchaser and multiple vendors, and (eventually) multiple buyers and multiple vendors. The latter two cases involve lot sizing as well as efficient allocation of orders. A number of models exist for determining such lot-sizing/allocation decisions. However, most of these are myopic in the sense that they tend to optimize the position of one party. The need here is for a broader approach, based on some joint criterion, that identifies those benefits which can be shared by all the parties concerned.

Finally, the practical problem with implementing a jointly optimal lot size in the current business environment should be discussed. Within the framework of an adversarial bargaining process between a buyer and a supplier, there is likely to be considerable reluctance on the part of either party to reveal the true values of its cost parameters. This can indeed be a formidable hurdle in establishing a joint optimal policy. In such a case, the party at a disadvantage can examine the other's ordering/setup-cost to carrying-charge ratio from the current or suggested lot size. Then, assuming identical carrying charges for both parties, the approximate values of the parameters α and β can be calculated. Thus, at least a "good" *JELS* and the potential advantages resulting from it may be determined. Once the other party is provided with this information and with the lure of a price adjustment, we are confident that traditional barriers quickly will dissolve and jointly optimal lot-sizing policies eventually will become common practice. [Received: July 20, 1984. Accepted: August 5, 1985.]

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